Reconstructing the expansion history of the Universe with a one-fluid approach

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Assuming that the Universe is filled by one single fluid, we present in the context of General Relativity a possible explanation for the acceleration of the Universe. We use ordinary thermodynamics and the fact that small matter perturbations barely propagate in our Universe, to derive a general solution for a single fluid in which the speed of sound vanishes. We find a model that contains ΛCDM as a special case, and is compatible with current observational data.

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To explain the (positive) acceleration of the Universe [1], it is usually assumed that, besides a dust-like fluid, the Universe is filled by an additional exotic fluid that accounts for about 75% of the energy density in the Universe. Many models are known in the literature that intend to describe the nature of this additional component [2, 3]; the simplest explanation is obtained by assuming the existence of a cosmological constant which is the basic ingredient of the well-known ΛCDM model. Despite its simplicity, the ΛCDM model suffers from various theoretical shortcomings [4], so it appears inadequate to be considered as a definitive model. In this work, we propose a different approach based on the assumption that the dynamics of the Universe satisfies the laws of ordinary thermodynamics and on the observational fact that the speed of sound vanishes for a matter fluid [5].

A perfectly homogeneous and isotropic cosmology with zero spatial curvature[1] is described by the Friedman-Robertson-Walker (FRW) line element

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 \sin^2\theta d\phi^2) \ . \qquad (1)$$

Moreover, the gravitational source is assumed to be described by the energy-momentum tensor of a perfect fluid [6]: $T_{\mu\nu} = \mathrm{diag}(\rho(t), -p(t), -p(t), -p(t))$. One can show that for a cosmological model satisfying the above symmetry conditions it is possible to apply the laws of ordinary thermodynamics in a consistent manner [7]. Let us consider a reversible and frictionless universe, satisfying the first law of thermodynamics, $dQ = dU + pdV$, and the equation of state (EoS) of an ideal gas $p = \rho(C_p - C_V)T$, where the heat capacities are defined as $C_V = \left(\frac{\partial U}{\partial T}\right)_V$ and $C_p = \left(\frac{\partial h}{\partial T}\right)_p$, with $h = U + pV$ being the enthalpy of the system. Furthermore, we assume that the evolution of the Universe is adiabatic and reversible so that a polytropic relation holds, $p = \tilde{p}\rho^\gamma$ [8], where $\gamma = C_p/C_V$. It is well known that currently the perturbations of the

density, $\delta\rho$, barely propagate in the Universe [5]. It then follows that the speed of sound $c_s^2 = \left(\frac{\partial p}{\partial \rho}\right)$ can be considered as vanishing. On the other hand, from the above thermodynamic assumptions we obtain for the Universe $c_s = \sqrt{\gamma(C_p - C_V)T}$ so that for a vanishing sound speed the allowed solutions are formally $C_p - C_V = 0$ or $\gamma = 0$. The first solution implies a pressureless universe whereas the second solution leads to a heat capacity $C_p = 0$, equivalent to $h = const$, and a pressure $p = -\rho C_V T$ which is negative for positive values of the heat capacity C_V.

We conclude that the important result of assuming an ideal-gas-like universe with zero speed of sound is that the pressure in that universe can be zero or negative[2]. Moreover, let us notice that this is a consequence of the fact that each particle of a given fluid undergoes an early isentropic process, when the wave amplitude is infinitesimal. This condition follows from the fact that, in general, the entropy is proportional to the square of the velocity and temperature gradients. We will now consider a vanishing speed of sound in the context of a FRW cosmology. Without putting any further information into the Einstein equations, we expect as a result a cosmological model in which the total pressure is negative and constant. Considering for the sake of simplicity the redshift z as the "time" variable defined by $dz/dt = -(1+z)H$, where t is the time coordinate, the conservation law for a generic fluid with EoS $p = w(z)\rho$, is

$$\frac{d\rho}{dz} - \frac{3(1+w)}{1+z}\rho = 0 \ . \qquad (2)$$

Assuming $c_s^2 = \partial p/\partial \rho = 0$, we get

$$\frac{dw}{dz} + \frac{3w(1+w)}{1+z} = 0 \ , \qquad (3)$$

whose solutions are

$$w = 0 \quad \text{and} \quad w = -\frac{1}{1 - \xi(1+z)^3} \ . \qquad (4)$$

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[1] We limit ourselves to this case to be in accordance with observations. The generalization to the case of nonzero spatial curvature is straightforward.

[2] Similar results were obtained by analyzing the ideal gas in the context of geometrothermodynamics [9]

Notice that as a consequence of the thermodynamic hypothesis presented above, we obtain in a straightforward manner a dust-like term and an additional term with a time–dependent barotropic factor. This is a result of our model which is otherwise usually postulated arbitrarily in cosmology [10].

Introducing the above solutions for w into Eq. (2), we obtain the most general density for *one fluid* satisfying the condition $c_s = 0$

$$\rho(z) = (\rho_m + \tilde{\rho}\xi)(1+z)^3 - \tilde{\rho}. \tag{5}$$

Notice that the term $\rho_m(1+z)^3$ corresponds to the solution $w = 0$. This general solution involves three constant parameters, namely, ξ, ρ_m and $\tilde{\rho}$.

Introducing $\rho(z)$ into the first Friedmann equation $H^2 = (8\pi G/3)\rho(z)$, we obtain the generic normalized Hubble rate, $E \equiv \frac{H}{H_0}$,

$$E = \sqrt{(\Omega_X + 1)(1+z)^3 - \Omega_X}, \tag{6}$$

where we adopt the convention $\Omega_X \equiv \frac{\tilde{\rho}}{\rho_c}$, with the critical density defined as $\rho_c \equiv \frac{3H_0^2}{8\pi G}$. In Eq. (6), we used the condition $E(z=0) = 1$, which gives

$$\xi = \frac{1 + \Omega_X - \Omega_m}{\Omega_X}, \tag{7}$$

where $\Omega_m \equiv \frac{\rho_m}{\rho_c}$. Therefore, to determine the Hubble rate $E(z)$ we need the constant Ω_X, whereas to determine the barotropic factor $w(z)$ in Eq. (4) it is necessary to know the two constants Ω_m and Ω_X.

Using the relationship as $\Omega_X = \frac{\Omega_m - 1}{1 - \xi}$ which follows from Eq. (7), it is possible to rewrite Eq. (6) as

$$E = \sqrt{\tilde{\Omega}_m(1+z)^3 + \tilde{\Omega}_\Lambda}, \tag{8}$$

where $\tilde{\Omega}_m \equiv \frac{\xi - \Omega_m}{\xi - 1}$ and the cosmological constant term reads $\tilde{\Omega}_\Lambda \equiv 1 - \tilde{\Omega}_m$.

The ΛCDM model is contained in Eq. (5) in the limiting case $\xi = 0$ with $\tilde{\rho} = \rho_m - 1$ so that ρ_m turns out to represent the sum of the baryonic and the cold dark matter densities. It follows that the model presented here is a generalization of ΛCDM and it is the result of the physical assumption that the Universe is made of only one matter fluid in which the the speed of sound is required to vanish. Hence, the crucial difference lies in the fact that in the present model no cosmological constant is postulated *a priori*.

In addition, difficulties related to the well-known problems of coincidence and of fine tuning [2] are solved in the context of the present model by the presence of a variable barotropic factor $w(z)$ and by the fact that no cosmological constant is assumed to be related to the vacuum energy.

For instance, if we consider $\Omega_m = 0.274$ and $w = -0.980$ [11], we get from Eqs. (4) and (7) $\xi \approx -0.02$

and $\Omega_X \approx -0.712$, which represents a value of Ω_X, compatible with an expanding Universe.

Moreover, it is possible to infer the limits of the evolution of $w(z)$ as

$$\begin{cases} w_0 = \frac{1}{\xi - 1}, & for \quad z = 0; \\[2mm] w_\infty = 0, & for \quad z \to \infty. \end{cases} \tag{9}$$

Eqs. (9) show that at redshift $z = 0$ w_0 is a constant which predicts $\xi \leq 0$ for $w \geq -1$. At higher redshift, the usual dust–like component dominates, because $w_\infty = 0$.

If the Universe accelerates, the so-called acceleration parameter, defined as

$$q = -1 - \frac{\dot{H}}{H^2}, \tag{10}$$

must be negative. From Eq. (10), for our model we obtain

$$q = -1 + \frac{3(1 + \Omega_X)(1+z)^3}{2 + 2(1 + \Omega_X)z\left[3 + z(3 + z)\right]}, \tag{11}$$

so that at $z = 0$ it reduces to

$$q_0 = \frac{1}{2}(1 + 3\Omega_X). \tag{12}$$

For the particular value $\Omega_X = -0.712$, we get $q_0 \approx -0.57$ which is in agreement with observations [12].

At the moment in which the acceleration starts $(q = 0)$, the correspondent redshift reads

$$z_{acc} = -1 + \frac{\left[2(-\Omega_X - 2\Omega_X^2 - \Omega_X^3)\right]^{\frac{1}{3}}}{1 + \Omega_X}, \tag{13}$$

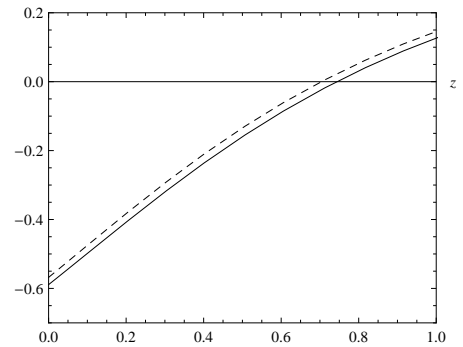so that for $\Omega_X = -0.712$ we have $z_{acc} \approx 0.7$.

FIG. 1: In this graphic is plotted $q(z)$ for our model (dashed line) and ΛCDM (black line). The indicative values are $\Omega_m = 0.274$, $\Omega_X = -0.712$.

Using Eqs. (6) and (8), it is possible to perform an experimental procedure to constrain the values of the constants Ω_m, Ω_X and ξ. In particular, we employ the three most common fitting procedures: Supernovae Ia

(SNeIa), Baryonic Acoustic Oscillation (BAO) and Cosmic Microwave Background (CMB). We will use of the most recent updated Union 2 compilation [11], which alleviates the problem of systematics.

Thus, associating to each Supernova modulus μ the corresponding 1σ error, denoted by σ_μ, we define the distance modulus $\mu = 25 + 5\log_{10}\frac{d_L}{Mpc}$, where $d_L(z)$ is the luminosity distance

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}, \qquad (14)$$

and we minimize the chi square, defined as follows

$$\chi_{SN}^2 = \sum_i \frac{(\mu_i^{\text{theor}} - \mu_i^{\text{obs}})^2}{\sigma_i^2}. \qquad (15)$$

The second test that we perform is related to the observations of large scale galaxy clusterings, which provide the signatures of the BAO [13]. We use the measurement of the peak of luminous red galaxies observed in Sloan Digital Sky Survey (SDSS), denoted by A

$$A = \sqrt{\Omega_m}\left[\frac{H_0}{H(z_{BAO})}\right]^{\frac{1}{3}}\left[\frac{1}{z_{BAO}}\int_0^{z_{BAO}}\frac{H_0}{H(z)}dz\right]^{\frac{2}{3}}, \qquad (16)$$

with $z_{BAO} = 0.35$. In addition, the observed A is estimated to be

$$A_{obs} = 0.469\left(\frac{0.95}{0.98}\right)^{-0.35}, \qquad (17)$$

with an error $\sigma_A = 0.017$. In the case of the BAO measurement we minimize the chi square

$$\chi_{BAO}^2 = \left(\frac{A - A_{obs}}{\sigma_A}\right)^2. \qquad (18)$$

Finally, for the CMB test we define the so-called CMB shift parameter

$$R = \sqrt{\Omega_m}\int_0^{z_{CMB}}\frac{H_0}{H(z)}dz, \qquad (19)$$

with $z_{CMB} = 1091.36$ [14]. It gives a complementary bound to the SNeIa data and BAO because the SNeIa redshift is $z < 2$, $z_{BAO} = 0.35$, while here $z \sim 1100$.

We minimize the chi square

$$\chi_{CMB}^2 = \left(\frac{R - R_{obs}}{\sigma_R}\right)^2. \qquad (20)$$

It is important to note that BAO and CMB do not depend on the values of H_0. We summarize the results of this numerical procedure in Tab. I

The values of Ω_m, Ω_X and ξ of Tab. I are in agreement with the theoretical results showed previously.

During the last decades, different parametrizations of $w(z)$ were proposed [2]. For instance, $w = w_1 + w_2 z$,

$\Omega_m(SN)$	$\Omega_m(BAO)$	$\Omega_m(CMB)$
0.275 ± 0.016	0.264 ± 0.016	0.280 ± 0.019
$\xi(SN)$	$\xi(BAO)$	$\xi(CMB)$
0.036 ± 0.009	-0.050 ± 0.011	-0.040 ± 0.012
$\Omega_X(SN)$	$\Omega_X(BAO)$	$\Omega_X(CMB)$
-0.765 ± 0.058	-0.700 ± 0.061	-0.729 ± 0.050

TABLE I: Summary of the numerical results for our model; the constants Ω_m and ξ have been fitted by using Eq. (8) and the parameter Ω_X by using Eq. (6). The chi square's are $\chi_{SNeIa}^2 = 1.010$, $\chi_{BAO}^2 = 0.989$, $\chi_{CMB}^2 = 1.000$, and $\chi_{SNeIa}^2 = 1.030$, $\chi_{BAO}^2 = 1.010$, $\chi_{CMB}^2 = 0.997$, respectively for the test with Eq. (6) and Eq. (8); the mean values for the three parameters are $\Omega_{m\,(mean)} = 0.273$, $\Omega_{X\,(mean)} = -0.731$ and $\xi_{mean} = -0.042$. Note that the value of H_0 for the SNeIa tests is $H_0 = 72 \pm 5\,Km\,s^{-1}\,Mpc^{-1}$.

$w = w_\alpha + w_\beta\log(1 + z)$ or $w = w_0 + w_a(1 - a)$. In particular, the third case was introduced by Chevallier, Polarski and Linder and it is referred to as the CPL parametrization [15]. The CPL parametrization has the advantages that at low and very high redshift it reduces to constant values, respectively $w(z \to 0) = w_0$, and $w(z \to \infty) = w_0 + w_a$. However, all the parametrizations suggested so far for $w(z)$ are either *ad hoc* proposals or the result of phenomenological assumptions only. Our model predicts a theoretical barotropic factor $w(z)$, and it is also able to reproduce previous results (see, for instance, [5]).

Moreover, the barotropic factor is also connected to an interesting quantity, that one might consider as a natural measure of time variation, namely, $\frac{dw}{d\ln(1+z)}\big|_{z=1}$. In models involving a scalar field φ with potential $V(\varphi)$, this quantity is related to the slow-roll potential $\propto \frac{V'}{V}$, in the region $z = 1$, where the scalar field is most likely to be evolving as the epoch of matter domination changes over to dark energy[3] domination. For the CPL parametrization $\frac{dw}{d\ln(1+z)}\big|_{z=1} = \frac{w_a}{2}$, while for the present model it is

$$\frac{dw}{d\ln(1+z)}\bigg|_{z=1} = -24\Omega_X\frac{1 + \Omega_X - \Omega_m}{(8 + 7\Omega_X - 8\Omega_m)^2}. \qquad (21)$$

By comparing our result with CPL we have two solutions. One of these solutions is physically compatible with the accelerating scenario; in fact, by considering $\Omega_{m(mean)}$ and the indicative value $w_a = 0.58$ [15], we find $\Omega_X \approx -0.716$, in agreement with the observational results.

We present below the graphics of the evolution of $w(z)$,

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[3] It is a common opinion to refer to the missing ingredient, driving the acceleration, as dark energy.

$q(z)$ and the expansion history for $a(t)$, given by [15, 16]

$$H_0 t(a) = \int_a^1 \frac{da'}{a' \, E(a')} \, . \tag{22}$$

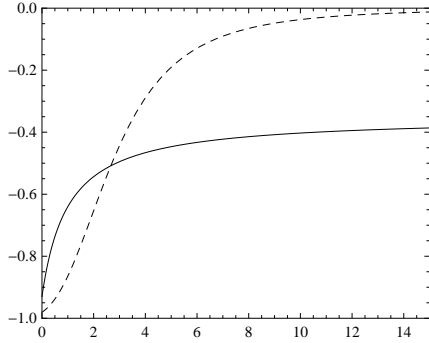FIG. 2: In this graphic is plotted $w(z)$ (Y axis) for our model (dashed line) and CPL (black line). The indicative values are $\Omega_m = 0.274$, $\Omega_X = -0.712$, $w_0 = -0.93$ and $w_a = 0.58$.
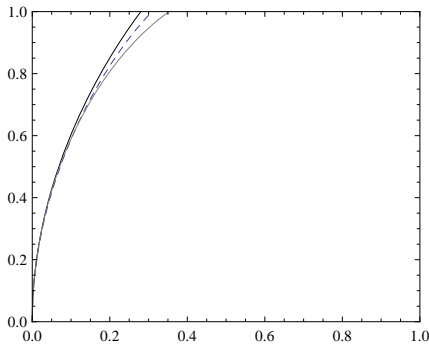

FIG. 3: In this graphic is plotted the expansion history of $a(t)$ (X axis) versus $H_0 t$ (Y axis) for our model (dashed line), ΛCDM (black line) and CPL (grey line). The indicative values are $\Omega_m = 0.274$, $\Omega_X = -0.712$, $w_0 = -0.93$ and $w_a = 0.58$.

The presence of baryonic and dark matter is generally intertwined with the addition of an exotic fluid which drives the acceleration. Unfortunately, all the attempts to describe this unexpected acceleration, suffer from various shortcomings. Moreover, the ΛCDM remains the favorite fitting model to describe the Universe dynamics, by including in Einstein equations a second fluid characterized by the cosmological constant. We showed that it is possible to discard the existence of a second fluid and, instead, to use ordinary thermodynamics. Assuming a vanishing speed of sound in order to guarantee that small matter perturbations do not propagate, we obtain a theoretical parametrization for w, which generalizes the ΛCDM model, reducing to it in a special case. In addition, the model presented here solves the coincidence and fine tuning problems in a straightforward manner. Using the cosmological tests of SNeIa, BAO and CMB, it was shown that our model is able to reproduce the observable Universe, and is in agreement with the theoretical limits.

Acknowledgements

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